GR

't Hooft Exercise 8.1 pg 37

$$\tilde{R}_{k\alpha} = g^{\lambda}_{\nu}\,\tilde{R}^{\nu}{}_{k\lambda\alpha} = R_{k\alpha} + D_\nu\,\delta\Gamma^{\nu}_{k\alpha} - D_\alpha\,\delta\Gamma^{\lambda}_{k\lambda}$$

$$D_\nu\,\delta\Gamma^{\nu}_{k\alpha} = \tfrac{1}{2} D_\nu \left\{ g^{\nu\varepsilon}\left[D_\alpha \delta g_{\varepsilon k} + D_k \delta g_{\varepsilon\alpha} - D_\varepsilon \delta g_{k\alpha} \right] \right\}$$

$$= \tfrac{1}{2} D_\nu \left[D_\alpha \delta g^{\nu}_{k} + D_k \delta g^{\nu}_{\alpha} - D^{\nu} \delta g_{k\alpha} \right]$$

$$D_\alpha\,\delta\Gamma^{\lambda}_{k\lambda} = \tfrac{1}{2} D_\alpha \left\{ g^{\lambda\varepsilon}\left[D_\lambda \delta g_{\varepsilon k} + D_k \delta g_{\varepsilon\lambda} - D_\varepsilon \delta g_{k\lambda} \right] \right\}$$

$$= \tfrac{1}{2} D_\alpha \left[D_\lambda \delta g^{\lambda}_{k} + D_k \delta g^{\lambda}_{\lambda} - D_\varepsilon \delta g^{\varepsilon}_{k} \right]$$

$$\Rightarrow \tilde{R}_{k\alpha} = R_{k\alpha} + \frac{1}{2}\left[-D^2 \delta g_{k\alpha} + D_\nu D_\alpha \delta g^{\nu}_{k} + D_\nu D_k \delta g^{\nu}_{\alpha} + D_\alpha D_k \delta g^{\lambda}_{\lambda} \right]$$

Swapping $k \to \mu$, $\alpha \to \nu$ gives (8.7).

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